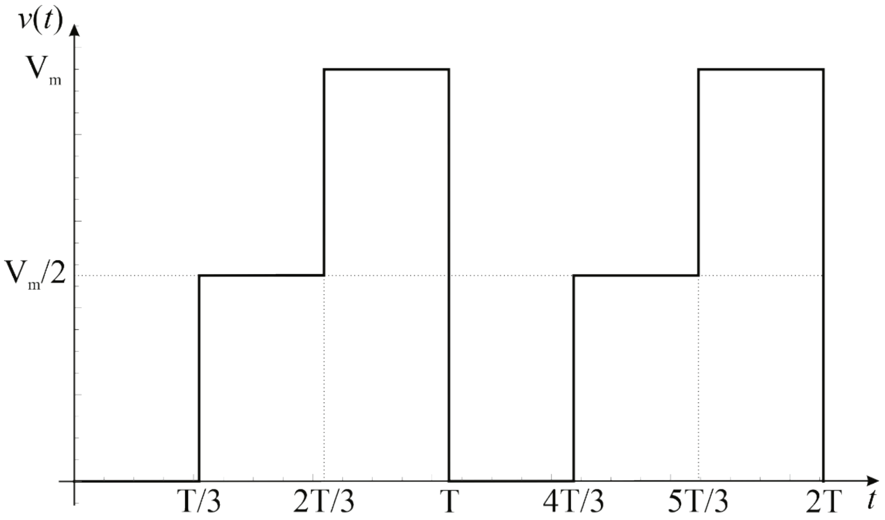
Srednja i efektivna vrednost signala, Furijeova transformacija

1. Izračunati srednju i efektivnu vrednost za signal dat na slici 1.



$$v(t)=\begin{cases}0, & 0\le \mathrm{t}<\mathrm{T}/3\\ \dfrac{V_{\mathrm{m}}}{2}, & \mathrm{T}/3\le \mathrm{t}<2\mathrm{T}/3\\ V_{\mathrm{m}}, & 2\mathrm{T}/3\le \mathrm{t}<\mathrm{T}\end{cases}$$

Slika 1.

Odgovor: Efektivna vrednost signala se izračunava prema formuli:

$$V_{\mathrm{ef}}=\sqrt{\frac{1}{\Delta t}\int_0^{\Delta t} v^2(t)\,dt}\qquad \text{ili}\qquad V_{\mathrm{ef}}=\sqrt{\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} v^2(t)\,dt}\,.$$

$$V_{\mathrm{ef}}=\sqrt{\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} v^2(t)\,dt}=\sqrt{\frac{1}{\mathrm{T}}\left(\int_0^{\mathrm{T}/3} v^2(t)\,dt+\int_{\mathrm{T}/3}^{2\mathrm{T}/3} v^2(t)\,dt+\int_{2\mathrm{T}/3}^{\mathrm{T}} v^2(t)\,dt\right)}=$$

$$=\sqrt{\frac{1}{\mathrm{T}}\left(\int_0^{\mathrm{T}/3} 0\cdot dt+\int_{\mathrm{T}/3}^{2\mathrm{T}/3}\left(\frac{V_{\mathrm{m}}}{2}\right)^2 dt+\int_{2\mathrm{T}/3}^{\mathrm{T}} V_{\mathrm{m}}^2\,dt\right)}=\sqrt{\frac{1}{\mathrm{T}}\left(0+\frac{V_{\mathrm{m}}^2}{4}t\Big|_{\mathrm{T}/3}^{2\mathrm{T}/3}+V_{\mathrm{m}}^2\,t\Big|_{2\mathrm{T}/3}^{\mathrm{T}}\right)}=$$

$$=\sqrt{\frac{1}{\mathrm{T}}\left(\frac{V_{\mathrm{m}}^2}{4}\left(\frac{2\mathrm{T}}{3}-\frac{\mathrm{T}}{3}\right)+V_{\mathrm{m}}^2\left(\mathrm{T}-\frac{2\mathrm{T}}{3}\right)\right)}=\sqrt{\frac{1}{\mathrm{T}}\cdot\frac{\mathrm{T}}{3}\left(\frac{V_{\mathrm{m}}^2}{4}+V_{\mathrm{m}}^2\right)}=\sqrt{\frac{1}{3}\cdot\frac{5V_{\mathrm{m}}^2}{4}}=\sqrt{\frac{5}{12}}\cdot V_{\mathrm{m}}$$

Srednja vrednost se izračunava prema formuli:

$$V_0=\frac{1}{\Delta t}\int_0^{\Delta t} v(t)\,dt\quad \text{ili}\qquad V_0=\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} v(t)\,dt\,.$$

$$V_0=\frac{1}{T}\int_0^{T} v(t)\,dt=\frac{1}{T}\left(\int_0^{T/3} v(t)\,dt+\int_{T/3}^{2T/3} v(t)\,dt+\int_{2T/3}^{T} v(t)\,dt\right)=$$

$$=\frac{1}{T}\left(\int_0^{T/3}0\cdot dt+\int_{T/3}^{2T/3}\frac{V_m}{2}\cdot dt+\int_{2T/3}^{T}V_m\,dt\right)=\frac{1}{T}\left(\frac{V_m}{2}\cdot t\Big|_{T/3}^{2T/3}+V_m\cdot t\Big|_{2T/3}^{T}\right)=$$

$$\frac{1}{T}\left(\frac{V_m}{2}\cdot\left(\frac{2T}{3}-\frac{T}{3}\right)+V_m\cdot\left(T-\frac{2T}{3}\right)\right)=\frac{1}{\cancel{T}}\left(\frac{V_m}{2}\cdot\frac{\cancel{T}}{3}+V_m\cdot\frac{\cancel{T}}{3}\right)=\frac{V_m}{2}.$$

2. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu A_k za periodični signal dat na slici 1.

Odgovor: Ortogonalna komponenta A_k harmonika k-tog reda je određena formulom:

$$A_k=\frac{2}{T}\int_0^{T} v(t)\cos k\omega t dt$$

Za dati signal je:

$$A_k=\frac{2}{T}\int_0^{T} v(t)\cos k\omega t dt=\frac{2}{T}\left(\int_0^{T/3}0\cdot\cos k\omega t dt+\int_{T/3}^{2T/3}\frac{V_m}{2}\cdot\cos k\omega t dt+\int_{2T/3}^{T}V_m\cdot\cos k\omega t dt\right)=$$

$$=\frac{V_m}{T}\int_{T/3}^{2T/3}\cos k\omega t dt+\frac{2V_m}{T}\int_{2T/3}^{T}\cos k\omega t dt=\frac{V_m}{k\omega\cdot T}\sin k\omega t\Big|_{T/3}^{2T/3}+\frac{2V_m}{k\omega\cdot T}\sin k\omega t\Big|_{2T/3}^{T}=$$

$$=\frac{V_m}{k\frac{2\pi}{\cancel{T}}\cdot\cancel{T}}\left(\sin k\frac{2\pi}{\cancel{T}}\cdot\frac{2\cancel{T}}{3}-\sin k\frac{2\pi}{\cancel{T}}\cdot\frac{\cancel{T}}{3}\right)+\frac{\cancel{2}V_m}{k\frac{\cancel{2}\pi}{\cancel{T}}\cdot\cancel{T}}\left(\sin k\frac{2\pi}{\cancel{T}}\cdot\cancel{T}-\sin k\frac{2\pi}{\cancel{T}}\cdot\frac{2\cancel{T}}{3}\right)=$$

$$=\frac{V_m}{2k\pi}\left(\sin\frac{4k\pi}{3}-\sin\frac{2k\pi}{3}\right)+\frac{V_m}{k\pi}\left(\cancel{\sin 2k\pi}-\sin\frac{4k\pi}{3}\right)=\frac{V_m}{2k\pi}\sin\frac{4k\pi}{3}-\frac{V_m}{k\pi}\sin\frac{4k\pi}{3}-\frac{V_m}{2k\pi}\sin\frac{2k\pi}{3}$$

$$=-\frac{V_m}{2k\pi}\left(\sin\frac{4k\pi}{3}+\sin\frac{2k\pi}{3}\right)=-\frac{V_m}{k\pi}\sin k\pi\cdot\cos\frac{k\pi}{3}=0\quad \text{za } k\in\mathbb{Z}.$$

3. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu B_k za periodični signal dat na slici 1.

Odgovor: Ortogonalna komponenta B_k harmonika k-tog reda je određena formulom:

$$B_k=\frac{2}{T}\int_0^{T} v(t)\sin k\omega t dt$$

Za dati signal je:

$$B_k=\frac{2}{T}\int_0^{T} v(t)\sin k\omega t dt=\frac{2}{T}\left(\int_0^{T/3}0\cdot\sin k\omega t dt+\int_{T/3}^{2T/3}\frac{V_m}{2}\cdot\sin k\omega t dt+\int_{2T/3}^{T}V_m\cdot\sin k\omega t dt\right)=$$

$$=\frac{V_m}{T}\int_{T/3}^{2T/3}\sin k\omega t dt+\frac{2V_m}{T}\int_{2T/3}^{T}\sin k\omega t dt=-\frac{V_m}{k\omega\cdot T}\cos k\omega t\Big|_{T/3}^{2T/3}-\frac{2V_m}{k\omega\cdot T}\cos k\omega t\Big|_{2T/3}^{T}=$$

$$=-\frac{V_m}{k\frac{2\pi}{T}\cdot T}\left(\cos k\frac{2\pi}{T}\cdot\frac{2T}{3}-\cos k\frac{2\pi}{T}\cdot\frac{T}{3}\right)-\frac{2V_m}{k\frac{2\pi}{T}\cdot T}\left(\cos k\frac{2\pi}{T}\cdot T-\cos k\frac{2\pi}{T}\cdot\frac{2T}{3}\right)=$$

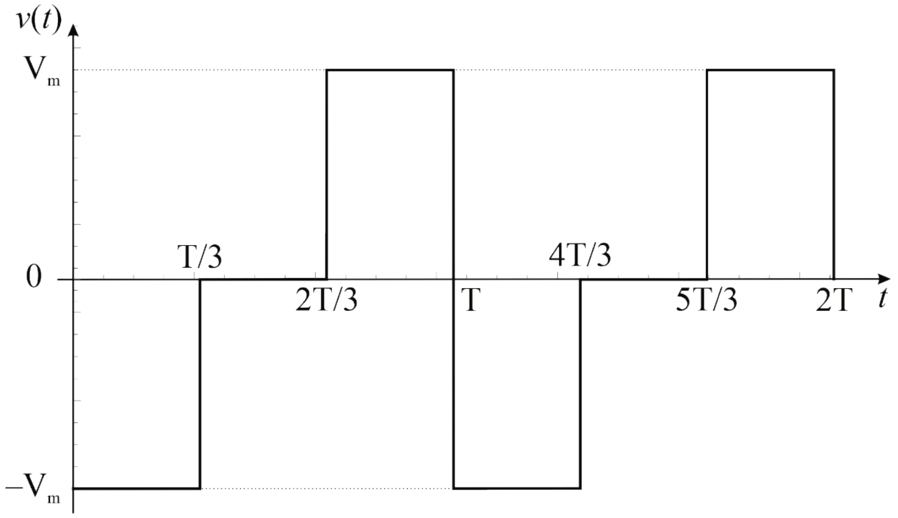
$$=-\frac{V_m}{2k\pi}\left(\cos\frac{4k\pi}{3}-\cos\frac{2k\pi}{3}\right)-\frac{V_m}{k\pi}\left(\cos 2k\pi-\cos\frac{4k\pi}{3}\right)=$$

$$=-\frac{V_m}{2k\pi}\cos\frac{4k\pi}{3}+\frac{V_m}{k\pi}\cos\frac{4k\pi}{3}+\frac{V_m}{2k\pi}\cos\frac{2k\pi}{3}-\frac{V_m}{k\pi}=$$

$$=\frac{V_m}{2k\pi}\left(\cos\frac{4k\pi}{3}+\cos\frac{2k\pi}{3}\right)-\frac{V_m}{k\pi}=\frac{V_m}{k\pi}\cos k\pi\cdot\cos\frac{k\pi}{3}-\frac{V_m}{k\pi}=\frac{V_m}{k\pi}\left(\cos k\pi\cdot\cos\frac{k\pi}{3}-1\right)\quad \text{za } k\in\mathbb{Z}$$

$$B_k=\begin{cases}0, & k\equiv 0\pmod 3\\ -\frac{3V_m}{2k\pi}, & k\equiv 1,2\pmod 3\end{cases}$$

4. Izračunati srednju i efektivnu vrednost za signal dat na slici 2.



$$v(t)=\begin{cases}-V_m, & 0\le t\le\frac{T}{3}\\ 0, & \frac{T}{3}<t<\frac{2T}{3}\\ V_m, & \frac{2T}{3}\le t<T\end{cases}$$

Slika 2.

Odgovor:

$$V_0=\frac{1}{T}\int_0^T v(t)dt=\frac{1}{T}\left(\int_0^{T/3}v(t)dt+\int_{T/3}^{2T/3}v(t)dt+\int_{2T/3}^{T}v(t)dt\right)=$$

$$=\frac{1}{T}\left(\int_0^{T/3}(-V_m)dt+\int_{T/3}^{2T/3}0\cdot dt+\int_{2T/3}^{T}dt\right)=\frac{1}{T}\left(-V_m\cdot t\Big|_0^{T/3}+0+V_m\cdot t\Big|_{2T/3}^{T}\right)=$$

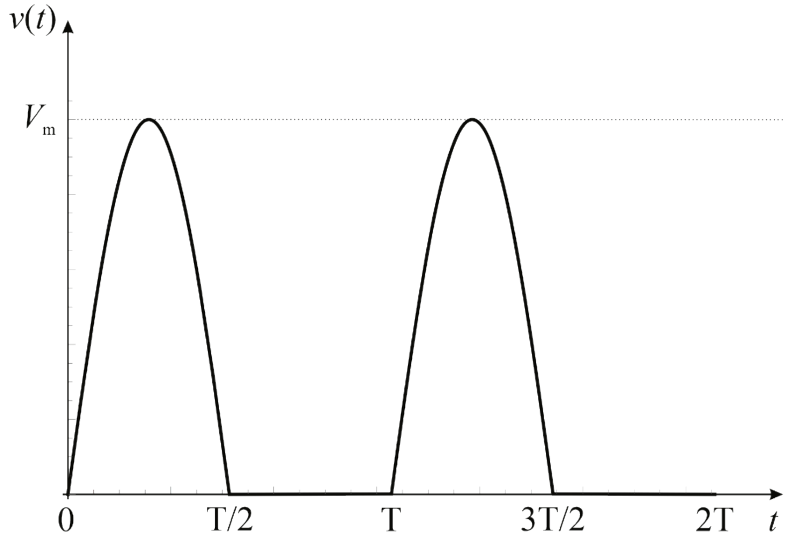
$$=\frac{1}{T}\left(-V_m\left(\frac{T}{3}-0\right)+V_m\left(T-\frac{2T}{3}\right)\right)=\frac{1}{T}\left(-V_m\cdot\frac{T}{3}+V_m\cdot\frac{T}{3}\right)=0.$$

$$V_{ef}=\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}=\sqrt{\frac{1}{T}\left(\int_0^{T/3} v^2(t)\,dt+\int_{T/3}^{2T/3} v^2(t)\,dt+\int_{2T/3}^{T} v^2(t)\,dt\right)}=$$

$$=\sqrt{\frac{1}{T}\left(\int_0^{T/3}(-V_m)^2\,dt+\int_{T/3}^{2T/3}0\cdot dt+\int_{2T/3}^{T}V_m^2\,dt\right)}=\sqrt{\frac{1}{T}\left(V_m^2\cdot t\Big|_0^{T/3}+0+V_m^2\cdot t\Big|_{2T/3}^{T}\right)}=$$

$$=\sqrt{\frac{1}{T}\left(V_m^2\left(\frac{T}{3}-0\right)+V_m^2\left(T-\frac{2T}{3}\right)\right)}=\sqrt{\frac{1}{T}\cdot\frac{2T}{3}\cdot V_m^2}=\sqrt{\frac{2}{3}}\cdot V_m$$

5. Izračunati srednju i efektivnu vrednost za signal dat na slici 3.



$$v(t)=\begin{cases} V_m\sin\omega t, & 0\le t\le \frac{T}{2}\\ 0, & \frac{T}{2}<t<T\end{cases}$$

Slika 3.

Odgovor:

$$V_0=\frac{1}{T}\int_0^T v(t)\,dt=\frac{1}{T}\int_0^{T/2} v(t)\,dt=$$

$$=\frac{1}{T}\int_0^{T/2}V_m\sin\omega t\cdot dt=\frac{V_m}{T\omega}\cos\omega t\Big|_0^{T/2}=-\frac{V_m}{\cancel{T}\cdot\frac{2\pi}{\cancel{T}}}\cdot\cos\frac{2\pi t}{T}\Big|_0^{T/2}=$$

$$=-\frac{V_m}{2\pi}\left(\cos\frac{\cancel{2}\pi}{\cancel{T}}\cdot\frac{\cancel{T}}{\cancel{2}}-\cos\frac{2\pi}{T}\cdot 0\right)=-\frac{V_m}{2\pi}(\cos\pi-\cos 0)=-\frac{V_m}{2\pi}(-1-1)=\frac{V_m}{\pi}$$

$$V_{ef}=\sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}=\sqrt{\frac{1}{T}\int_0^{T/2}V_m^2\sin^2\omega t\,dt+\frac{1}{T}\int_{T/2}^{T}0\cdot dt}=\sqrt{\frac{V_m^2}{T}\int_0^{T/2}\sin^2\omega t\,dt}=$$

$$=\sqrt{\frac{V_m^2}{T}\int_0^{T/2}\frac{1-\cos 2\omega t}{2}dt}=\sqrt{\frac{V_m^2}{2T}\int_0^{T/2}dt-\frac{V_m^2}{2T}\int_0^{T/2}\cos 2\omega t\,dt}=$$

$$=\sqrt{\frac{V_m^2}{2T}t\Big|_0^{T/2}-\frac{V_m^2}{4\omega T}\sin 2\omega t\Big|_0^{T/2}}=\sqrt{\frac{V_m^2}{2T}\left(\frac{T}{2}-0\right)-\frac{V_m^2}{4\omega T}\left(\sin 2\omega\frac{T}{2}-0\right)}=$$

$$=\sqrt{\frac{V_m^2}{2\not{T}}\frac{\not{T}}{2}-\frac{V_m^2}{4\cdot\frac{2\pi}{\not{T}}\cdot\not{T}}\left(\sin 2\cdot\frac{\not{2}\pi}{\not{T}}\cdot\frac{\not{T}}{\not{2}}-0\right)}=\sqrt{\frac{V_m^2}{4}-\frac{V_m^2}{8\pi}\sin 2\pi}=\sqrt{\frac{V_m^2}{4}}=\frac{V_m}{2}$$

smena: $\omega=\frac{2\pi}{T}$, $\sin^2\omega t=\frac{1-\cos 2\omega t}{2}$

6. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu A_k za periodični signal dat na slici 3.

Odgovor: Ortogonalna komponenta A_k harmonika k-tog reda je određena formulom:

$$A_k=\frac{2}{T}\int_0^T v(t)\cos k\omega t dt$$

Za dati signal je:

$$A_k=\frac{2}{T}\int_0^T v(t)\cos k\omega t dt=\frac{2}{T}\left(\int_0^{T/2} V_m\cdot\sin\omega t\cdot\cos k\omega t dt+\int_{T/2}^{T}0\cdot dt\right)=$$

$$=\frac{2V_m}{T}\int_0^{T/2}\sin\omega t\cdot\cos k\omega t dt=\frac{V_m}{T}\int_0^{T/2}\left(\sin(\omega t+k\omega t)+\sin(\omega t-k\omega t)\right)dt=$$

$$=\frac{V_m}{T}\int_0^{T/2}\sin(k+1)\omega t dt-\frac{V_m}{T}\int_0^{T/2}\sin(k-1)\omega t dt=$$

$$=-\frac{V_m}{T(k+1)\omega}\cos((k+1)\omega t)\Big|_0^{\frac{T}{2}}+\frac{V_m}{T(k-1)\omega}\cos((k-1)\omega t)\Big|_0^{\frac{T}{2}}=$$

$$=-\frac{V_m}{\not{T}(k+1)\frac{2\pi}{\not{T}}}\cos\left((k+1)\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{2}}+\frac{V_m}{\not{T}(k-1)\frac{2\pi}{\not{T}}}\cos\left((k-1)\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{2}}=$$

$$=-\frac{V_m}{\not{T}(k+1)\frac{2\pi}{\not{T}}}\left(\cos\left((k+1)\frac{\not{2}\pi}{\not{T}}\frac{\not{T}}{\not{2}}\right)-1\right)+\frac{V_m}{\not{T}(k-1)\frac{2\pi}{\not{T}}}\left(\cos\left((k-1)\frac{\not{2}\pi}{\not{T}}\frac{\not{T}}{\not{2}}\right)-1\right)=$$

$$=\frac{V_m}{2\pi}\left(\frac{\cos(k-1)\pi-1}{k-1}-\frac{\cos(k+1)\pi-1}{k+1}\right)=\begin{cases}0, & k\text{ neparno}\\ -\frac{2V_m}{(k^2-1)\pi}, & k\text{ parno}\end{cases}$$

7. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu B_k za periodični signal dat na slici 3.

Odgovor: Ortogonalna komponenta A_k harmonika k-tog reda je određena formulom:

$$B_k = \frac{2}{\mathrm{T}} \int_0^{\mathrm{T}} v(t) \sin k\omega t dt$$

Za dati signal je:

$$B_k = \frac{2}{\mathrm{T}} \int_0^{\mathrm{T}} v(t) \sin k\omega t dt = \frac{2}{\mathrm{T}} \left(\int_0^{\mathrm{T}/2} V_{\mathrm{m}} \cdot \sin \omega t \cdot \sin k\omega t dt + \int_{\mathrm{T}/2}^{\mathrm{T}} 0 \cdot dt \right) =$$

$$= \frac{2V_{\mathrm{m}}}{\mathrm{T}} \int_0^{\mathrm{T}/2} \sin \omega t \cdot \sin k\omega t dt = \frac{V_{\mathrm{m}}}{\mathrm{T}} \int_0^{\mathrm{T}/2} \left(\cos(\omega t - k\omega t) - \cos(\omega t + k\omega t) \right) dt =$$

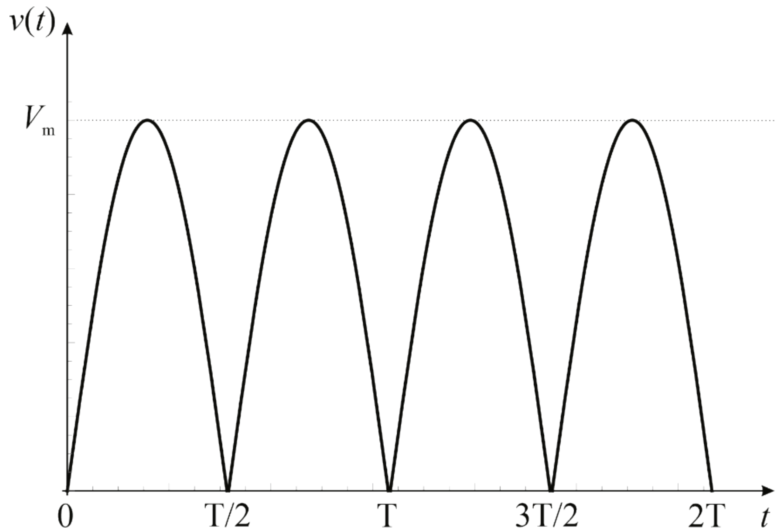
$$= \frac{V_{\mathrm{m}}}{\mathrm{T}} \int_0^{\mathrm{T}/2} \cos(k-1)\omega t dt - \frac{V_{\mathrm{m}}}{\mathrm{T}} \int_0^{\mathrm{T}/2} \cos(k+1)\omega t dt =$$

$$= \frac{V_{\mathrm{m}}}{\mathrm{T}(k-1)\frac{2\pi}{\mathrm{T}}} \sin\left((k-1)\frac{2\pi}{\mathrm{T}} t \right) \Bigg|_0^{\frac{\mathrm{T}}{2}} - \frac{V_{\mathrm{m}}}{\mathrm{T}(k+1)\frac{2\pi}{\mathrm{T}}} \sin\left((k+1)\frac{2\pi}{\mathrm{T}} t \right) \Bigg|_0^{\frac{\mathrm{T}}{2}} =$$

$$= \frac{V_{\mathrm{m}}}{\mathrm{T}(k-1)\frac{2\pi}{\mathrm{T}}} \sin\left((k-1)\frac{2\pi}{\mathrm{T}} \frac{\mathrm{T}}{2} \right) - \frac{V_{\mathrm{m}}}{\mathrm{T}(k+1)\frac{2\pi}{\mathrm{T}}} \sin\left((k+1)\frac{2\pi}{\mathrm{T}} \frac{\mathrm{T}}{2} \right) =$$

$$= \frac{V_{\mathrm{m}}}{2} \left(\frac{\sin(k-1)\pi}{(k-1)\pi} - \frac{\sin(k+1)\pi}{(k+1)\pi} \right) = \begin{cases} \frac{V_{\mathrm{m}}}{2}, & k=1 \\ 0, & k>1 \end{cases} \quad \left(\lim_{x\to 0} \frac{\sin x}{x} = 1 \right)$$

8. Izračunati srednju i efektivnu vrednost za signal dat na slici 4.



Slika 4.

$$v(t) = \begin{cases} V_{\mathrm{m}} \sin \omega t, & 0 \le t < \frac{\mathrm{T}}{2} \\ -V_{\mathrm{m}} \sin \omega t, & \frac{\mathrm{T}}{2} \le t < \mathrm{T} \end{cases}$$

Odgovor:

$$V_0=\frac{1}{T}\int_0^T v(t)dt=\frac{2}{T}\int_0^{T/2} v(t)dt=$$

$$=\frac{2}{T}\int_0^{T/2} V_m \sin\omega t\cdot dt=\frac{2V_m}{T\omega}\cos\omega t\Big|_0^{T/2}=-\frac{2V_m}{T\cdot\frac{2\pi}{T}}\cdot\cos\frac{2\pi t}{T}\Big|_0^{T/2}=$$

$$=-\frac{V_m}{\pi}\left(\cos\frac{2\pi}{T}\cdot\frac{T}{2}-\cos\frac{2\pi}{T}\cdot 0\right)=-\frac{V_m}{\pi}(\cos\pi-\cos 0)=-\frac{V_m}{\pi}(-1-1)=\frac{2V_m}{\pi}$$

$$V_{ef}=\sqrt{\frac{1}{T}\int_0^T v^2(t)dt}=\sqrt{\frac{2}{T}\int_0^{T/2} V_m^2\sin^2\omega t\,dt}=\sqrt{\frac{2V_m^2}{T}\int_0^{T/2}\sin^2\omega t\,dt}=$$

$$=\sqrt{\frac{2V_m^2}{T}\int_0^{T/2}\frac{1-\cos 2\omega t}{2}dt}=\sqrt{\frac{V_m^2}{T}\int_0^{T/2}dt-\frac{V_m^2}{T}\int_0^{T/2}\cos 2\omega t\,dt}=$$

$$=\sqrt{\frac{V_m^2}{T}t\Big|_0^{T/2}-\frac{V_m^2}{2\omega T}\sin 2\omega t\Big|_0^{T/2}}=\sqrt{\frac{V_m^2}{T}\left(\frac{T}{2}-0\right)-\frac{V_m^2}{2\omega T}\left(\sin 2\omega\frac{T}{2}-0\right)}=$$

$$=\sqrt{\frac{V_m^2}{T}\frac{T}{2}-\frac{V_m^2}{2\cdot\frac{2\pi}{T}\cdot T}\left(\sin 2\cdot\frac{2\pi}{T}\cdot\frac{T}{2}-0\right)}=\sqrt{\frac{V_m^2}{2}-\frac{V_m^2}{4\pi}\sin 2\pi}=\sqrt{\frac{V_m^2}{2}}=\frac{V_m}{\sqrt{2}}$$

9. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu A_k za periodični signal dat na slici 4.

Odgovor:

Kako je signal na slici 4 periodičan sa periodom $\frac{T}{2}$, u izrazu za A_k treba zameniti vrednosti za periodu i kružnu frekvenciju ($T\to\frac{T}{2},\quad \omega\to 2\omega$).

$$A_k=\frac{4}{T}\int_0^{T/2} v(t)\cdot\cos 2k\omega t\cdot dt=\frac{4}{T}\int_0^{T/2} V_m\cdot\sin\omega t\cdot\cos 2k\omega t dt=$$

$$=\frac{4V_m}{T}\int_0^{T/2}\sin\omega t\cdot\cos 2k\omega t dt=\frac{2V_m}{T}\int_0^{T/2}\left(\sin(\omega t+2k\omega t)+\sin(\omega t-2k\omega t)\right)dt=$$

$$=\frac{2V_m}{T}\int_0^{T/2}\sin(2k+1)\omega t dt-\frac{2V_m}{T}\int_0^{T/2}\sin(2k-1)\omega t dt=$$

$$=-\frac{2V_m}{T(2k+1)\omega}\cos((2k+1)\omega t)\Big|_0^{\frac{T}{2}}+\frac{2V_m}{T(2k-1)\omega}\cos((2k-1)\omega t)\Big|_0^{\frac{T}{2}}=$$

$$=-\frac{2V_{\mathrm{m}}}{T(2k+1)\frac{2\pi}{T}}\cos\left((2k+1)\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{2}}+\frac{2V_{\mathrm{m}}}{T(2k-1)\frac{2\pi}{T}}\cos\left((2k-1)\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{2}}=$$

$$=-\frac{V_{\mathrm{m}}}{T(2k+1)\frac{\pi}{T}}\left(\cos\left((2k+1)\frac{2\pi}{T}\frac{T}{2}\right)-1\right)+\frac{V_{\mathrm{m}}}{T(2k-1)\frac{\pi}{T}}\left(\cos\left((2k-1)\frac{2\pi}{T}\frac{T}{2}\right)-1\right)=$$

$$=\frac{V_{\mathrm{m}}}{\pi}\left(\frac{\cos(2k-1)\pi-1}{2k-1}-\frac{\cos(2k+1)\pi-1}{2k+1}\right)=-\frac{4V_{\mathrm{m}}}{(4k^2-1)\pi}$$

10. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu B_{k} za periodični signal dat na slici 4.

Odgovor:

Kako je signal na slici 4 periodičan sa periodom $\frac{T}{2}$, u izrazu za B_{k} treba zameniti vrednosti za periodu i kružnu frekvenciju ($T\rightarrow\frac{T}{2},\quad \omega\rightarrow 2\omega$).

$$B_k=\frac{4}{T}\int_0^{T/2}v(t)\cdot\sin 2k\omega t\cdot dt=\frac{4}{T}\int_0^{T/2}V_{\mathrm{m}}\cdot\sin\omega t\cdot\sin 2k\omega t dt=$$

$$=\frac{2V_{\mathrm{m}}}{T}\int_0^{T/2}\left(\cos(\omega t-2k\omega t)-\cos(\omega t+2k\omega t)\right)dt=$$

$$=\frac{2V_{\mathrm{m}}}{T}\int_0^{T/2}\cos(2k-1)\omega t dt-\frac{2V_{\mathrm{m}}}{T}\int_0^{T/2}\cos(2k+1)\omega t dt=$$

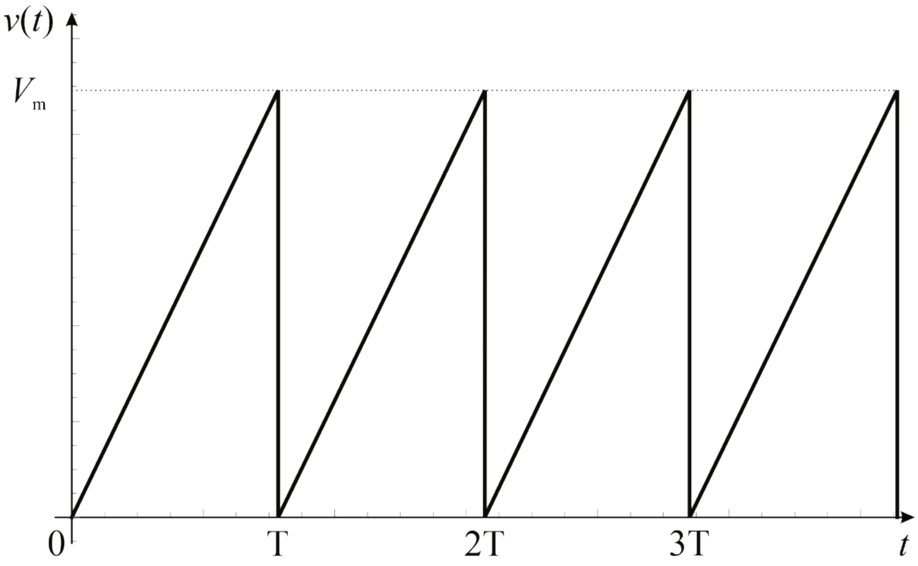
$$=\frac{2V_{\mathrm{m}}}{T(2k-1)\omega}\sin\left((2k-1)\omega t\right)\Big|_0^{\frac{T}{2}}-\frac{2V_{\mathrm{m}}}{T(2k+1)\omega}\sin\left((2k+1)\omega t\right)\Big|_0^{\frac{T}{2}}=$$

$$=\frac{2V_{\mathrm{m}}}{T(2k-1)\frac{2\pi}{T}}\sin\left((2k-1)\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{2}}-\frac{2V_{\mathrm{m}}}{T(2k+1)\frac{2\pi}{T}}\sin\left((2k+1)\frac{2\pi}{T}t\right)\Bigg|_0^{\frac{T}{2}}=$$

$$=\frac{V_{\mathrm{m}}}{(2k-1)\pi}\sin\left((2k-1)\frac{2\pi}{T}\frac{T}{2}\right)-\frac{V_{\mathrm{m}}}{(2k+1)\pi}\sin\left((2k+1)\frac{2\pi}{T}\frac{T}{2}\right)=$$

$$=V_{\mathrm{m}}\left(\frac{\sin(2k-1)\pi}{(2k-1)\pi}-\frac{\sin(2k+1)\pi}{(2k+1)\pi}\right)=0$$

11. Izračunati srednju i efektivnu vrednost za signal dat na slici 5.



$$v(t) = \frac{V_\mathrm{m}}{\mathrm{T}} \cdot t, \quad 0 \le t \le \mathrm{T}$$

Slika 5.

Odgovor :

$$V_0 = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} v(t)\,dt = \frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} \frac{V_\mathrm{m}}{\mathrm{T}} \cdot t \cdot dt = \frac{V_\mathrm{m}}{\mathrm{T}^2}\int_0^{\mathrm{T}} t \cdot dt = \frac{V_\mathrm{m}}{2\mathrm{T}^2} t^2 \Big|_0^{\mathrm{T}} = \frac{V_\mathrm{m}}{2\mathrm{T}^2}\left(\mathrm{T}^2 - 0\right) = \frac{V_\mathrm{m}}{2}$$

$$V_\mathrm{ef} = \sqrt{\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} v^2(t)\,dt} = \sqrt{\frac{1}{\mathrm{T}}\int_0^{\mathrm{T}} \frac{V_\mathrm{m}^2}{\mathrm{T}^2} \cdot t^2 \cdot dt} = \sqrt{\frac{V_\mathrm{m}^2}{\mathrm{T}^3}\int_0^{\mathrm{T}} t^2 \cdot dt} = \sqrt{\frac{V_\mathrm{m}^2}{3\mathrm{T}^3} t^3 \Big|_0^{\mathrm{T}}} = \sqrt{\frac{V_\mathrm{m}^2}{3\mathrm{T}^3}\left(\mathrm{T}^3 - 0\right)} = \frac{V_\mathrm{m}}{\sqrt{3}}$$

12. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu A_k za periodični signal dat na slici 5.

Odgovor:

$$A_k = \frac{2}{\mathrm{T}}\int_0^{\mathrm{T}} v(t)\cos k\omega t\,dt = \frac{2}{\mathrm{T}}\int_0^{\mathrm{T}} \frac{V_\mathrm{m}}{\mathrm{T}} \cdot t \cdot \cos k\omega t \cdot dt = \frac{2V_\mathrm{m}}{\mathrm{T}^2}\int_0^{\mathrm{T}} t \cdot \cos k\omega t \cdot dt = \frac{2V_\mathrm{m}}{\mathrm{T}^2 \cdot k\omega}\int_0^{\mathrm{T}} t \cdot d\left(\sin k\omega t\right) =$$

$$= \frac{2V_\mathrm{m}}{\mathrm{T}^2 \cdot k\omega}\left(t \sin k\omega t\Big|_0^{\mathrm{T}} - \int_0^{\mathrm{T}} \sin k\omega t \cdot dt \right) = \frac{\cancel{2}V_\mathrm{m}}{\mathrm{T}^{\cancel{2}} \cdot k\,\frac{\cancel{2}\pi}{\cancel{\mathrm{T}}}}\left(\mathrm{T}\sin k\frac{2\pi}{\cancel{\mathrm{T}}}\cancel{\mathrm{T}} - \int_0^{\mathrm{T}} \sin k\omega t \cdot dt \right) =$$

$$= \frac{V_\mathrm{m}}{k\pi\mathrm{T}}\left(\cancel{\mathrm{T}\sin 2k\pi} + \frac{1}{k\omega}\cos k\omega t \Big|_0^{\mathrm{T}} \right) = \frac{V_\mathrm{m}}{k\pi\cancel{\mathrm{T}}} \cdot \frac{1}{k\,\frac{2\pi}{\cancel{\mathrm{T}}}}\left(\cos k\omega\mathrm{T} - 1\right) = \frac{V_\mathrm{m}}{2k^2\pi^2}\left(\cos k\frac{2\pi}{\cancel{\mathrm{T}}}\cancel{\mathrm{T}} - 1\right) =$$

$$= \frac{V_\mathrm{m}}{2k^2\pi^2}\left(\cos 2k\pi - 1\right) = 0$$

(u četvrtom i petom koraku se koristi metod parcijalne integracije)

13. Primenom Furijeove transformacije, pronaći ortogonalnu komponentu B_k za periodični signal dat na slici 5.

Odgovor:

$$B_k = \frac{2}{\mathrm{T}}\int_0^{\mathrm{T}} v(t)\sin k\omega t dt = \frac{2}{\mathrm{T}}\int_0^{\mathrm{T}} \frac{V_\mathrm{m}}{\mathrm{T}}\cdot t\cdot \sin k\omega t\cdot dt = \frac{2V_\mathrm{m}}{\mathrm{T}^2}\int_0^{\mathrm{T}} t\cdot\sin k\omega t\cdot dt =$$

$$= -\frac{2V_\mathrm{m}}{\mathrm{T}^2\cdot k\omega}\int_0^{\mathrm{T}} t\cdot d\left(\cos k\omega t\right) = -\frac{2V_\mathrm{m}}{\mathrm{T}^2\cdot k\omega}\left(t\cos k\omega t\Big|_0^{\mathrm{T}} - \int_0^{\mathrm{T}} \cos k\omega t\cdot dt\right) =$$

$$= -\frac{2V_\mathrm{m}}{\mathrm{T}^2\cdot k\frac{2\pi}{\mathrm{T}}}\left(\mathrm{T}\cos k\frac{2\pi}{\mathrm{T}}\mathrm{T} - \int_0^{\mathrm{T}}\cos k\omega t\cdot dt\right) = -\frac{V_\mathrm{m}}{k\pi\mathrm{T}}\left(\mathrm{T}\cos 2k\pi - \frac{1}{k\omega}\sin k\omega t\Big|_0^{\mathrm{T}}\right) =$$

$$= -\frac{V_\mathrm{m}}{k\pi\mathrm{T}}\left(\mathrm{T} - \frac{1}{k\frac{2\pi}{\mathrm{T}}}\sin k\omega\mathrm{T}\right) = -\frac{V_\mathrm{m}}{k\pi\mathrm{T}}\left(\mathrm{T} - \frac{\mathrm{T}}{2k\pi}\sin k\frac{2\pi}{\mathrm{T}}\mathrm{T}\right) =$$

$$= -\frac{V_\mathrm{m}}{k\pi\mathrm{T}}\left(\mathrm{T} - \frac{\mathrm{T}}{2k\pi}\sin 2k\pi\right) = -\frac{V_\mathrm{m}}{k\pi}.$$

(u četvrtom i petom koraku se koristi metod parcijalne integracije)